EXAMPLE 9 Simplify: **(a)** $\log_3 3^7$; **(b)** $\log_{10} 10^{-5.2}$.

SOLUTION

a) $\log_3 3^7 = 7$ 7 is the exponent to which you raise 3 in order to get 3^7.

b) $\log_{10} 10^{-5.2} = -5.2$

Try Exercise 65.

We summarize the properties covered in this section as follows.

For any positive numbers M, N, and a $(a \neq 1)$:

$$\log_a (MN) = \log_a M + \log_a N; \qquad \log_a M^p = p \cdot \log_a M;$$

$$\log_a \frac{M}{N} = \log_a M - \log_a N; \qquad \log_a a^k = k.$$

CAUTION! Keep in mind that, in general,

$$\log_a (M + N) \neq \log_a M + \log_a N,$$
$$\log_a (M - N) \neq \log_a M - \log_a N,$$
$$\log_a (MN) \neq (\log_a M)(\log_a N),$$
$$\log_a \frac{M}{N} \neq \frac{\log_a M}{\log_a N}.$$

9.4 Exercise Set

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Concept Reinforcement *In each of Exercises 1–6, match the expression with an equivalent expression from the column on the right.*

1. (e)	$\log_7 20$	**a)**	$\log_7 5 - \log_7 4$
2. (f)	$\log_7 5^4$	**b)**	1
3. (a)	$\log_7 \frac{5}{4}$	**c)**	0
4. (b)	$\log_7 7$	**d)**	$\log_7 30$
5. (c)	$\log_7 1$	**e)**	$\log_7 5 + \log_7 4$
6. (d)	$\log_7 5 + \log_7 6$	**f)**	$4 \log_7 5$

Express as an equivalent expression that is a sum of logarithms.

7. $\log_3 (81 \cdot 27)$ $\log_3 81 + \log_3 27$

8. $\log_2 (16 \cdot 32)$ $\log_2 16 + \log_2 32$

9. $\log_4 (64 \cdot 16)$ $\log_4 64 + \log_4 16$

10. $\log_5 (25 \cdot 125)$ $\log_5 25 + \log_5 125$

11. $\log_c (rst)$ $\log_c r + \log_c s + \log_c t$

12. $\log_t (3ab)$ $\log_t 3 + \log_t a + \log_t b$

Express as an equivalent expression that is a single logarithm.

13. $\log_a 5 + \log_a 14$ $\log_a (5 \cdot 14)$, or $\log_a 70$

14. $\log_b 65 + \log_b 2$ $\log_b (65 \cdot 2)$, or $\log_b 130$

15. $\log_c t + \log_c y$ $\log_c (t \cdot y)$

16. $\log_t H + \log_t M$ $\log_t (H \cdot M)$

Express as an equivalent expression that is a product.

17. $\log_a r^8$ $8 \log_a r$

18. $\log_b t^5$ $5 \log_b t$

19. $\log_2 y^{1/3}$ $\frac{1}{3} \log_2 y$

20. $\log_{10} x^{1/2}$ $\frac{1}{2} \log_{10} x$

21. $\log_b C^{-3}$ $-3 \log_b C$

22. $\log_c M^{-5}$ $-5 \log_c M$

Express as an equivalent expression that is a difference of two logarithms.

23. $\log_2 \frac{25}{13}$ $\log_2 25 - \log_2 13$

24. $\log_3 \frac{23}{9}$ $\log_3 23 - \log_3 9$

25. $\log_b \frac{m}{n}$ $\log_b m - \log_b n$

26. $\log_a \frac{y}{x}$ $\log_a y - \log_a x$

Express as an equivalent expression that is a single logarithm.

27. $\log_a 17 - \log_a 6$ $\log_a \frac{17}{6}$

28. $\log_b 32 - \log_b 7$

29. $\log_b 36 - \log_b 4$

30. $\log_a 26 - \log_a 2$

31. $\log_a x - \log_a y$ $\log_a \frac{x}{y}$

32. $\log_b c - \log_b d$ $\log_b \frac{c}{d}$

Express as an equivalent expression, using the individual logarithms of w, x, y, and z.

33. $\log_a (xyz)$

34. $\log_a (wxy)$

35. $\log_a (x^3z^4)$

36. $\log_a (x^2y^5)$

37. $\log_a (x^2y^{-2}z)$

38. $\log_a (xy^2z^{-3})$

39. $\log_a \dfrac{x^4}{y^3z}$

40. $\log_a \dfrac{x^4}{yz^2}$

41. $\log_b \dfrac{xy^2}{wz^3}$

42. $\log_b \dfrac{w^2x}{y^3z}$

43. $\log_a \sqrt{\dfrac{x^7}{y^5z^8}}$

44. $\log_c \sqrt[3]{\dfrac{x^4}{y^3z^2}}$

45. $\log_a \sqrt[3]{\dfrac{x^6y^3}{a^2z^7}}$

46. $\log_a \sqrt[4]{\dfrac{x^8y^{12}}{a^3z^5}}$

Express as an equivalent expression that is a single logarithm and, if possible, simplify.

47. $8 \log_a x + 3 \log_a z$ $\log_a (x^8z^3)$

48. $2 \log_b m + \frac{1}{2} \log_b n$ $\log_b (m^2n^{1/2})$, or $\log_b (m^2\sqrt{n})$

49. $\log_a x^2 - 2 \log_a \sqrt{x}$ $\log_a x$

50. $\log_a \dfrac{a}{\sqrt{x}} - \log_a \sqrt{ax}$ $\log_a \dfrac{\sqrt{a}}{x}$

51. $\frac{1}{2} \log_a x + 5 \log_a y - 2 \log_a x$ $\log_a \dfrac{y^5}{x^{3/2}}$

52. $\log_a (2x) + 3(\log_a x - \log_a y)$ $\log_a \dfrac{2x^4}{y^3}$

53. $\log_a (x^2 - 4) - \log_a (x + 2)$ $\log_a (x - 2)$

54. $\log_a (2x + 10) - \log_a (x^2 - 25)$ $\log_a \dfrac{2}{x - 5}$

Given $\log_b 3 = 0.792$ and $\log_b 5 = 1.161$. If possible, use the properties of logarithms to calculate numerical values for each of the following.

55. $\log_b 15$ 1.953

56. $\log_b \frac{5}{3}$ 0.369

57. $\log_b \frac{3}{5}$ −0.369

58. $\log_b \frac{1}{3}$ −0.792

59. $\log_b \frac{1}{5}$ −1.161

60. $\log_b \sqrt{b}$ $\frac{1}{2}$

61. $\log_b \sqrt{b^3}$ $\frac{3}{2}$

62. $\log_b (3b)$ 1.792

63. $\log_b 8$ Cannot be found

64. $\log_b 45$ 2.745

Simplify.

65. $\log_t t^7$ 7

66. $\log_p p^4$ 4

67. $\log_e e^m$ m

68. $\log_Q Q^{-2}$ −2

Use the properties of logarithms to find each of the following.

$\log_3 9 + \log_3 81 = 6$

Aha! **69.** $\log_5 (125 \cdot 625)$ $\log_5 125 + \log_5 625 = 7$

70. $\log_3 (9 \cdot 81)$

71. $\log_2 16^5$ $5 \cdot \log_2 16 = 20$

72. $\log_3 27^7$ $7 \cdot \log_3 27 = 21$

TW **73.** Explain the difference between the phrases "the logarithm of a quotient" and "a quotient of logarithms."

TW **74.** How could you convince someone that

$$\log_a c \neq \log_c a?$$

SKILL REVIEW

To prepare for Section 9.5, review graphing functions and finding domains of functions.

Graph.

75. $f(x) = \sqrt{x} - 3$ [7.1]

76. $g(x) = \sqrt[3]{x} + 1$ [7.1]

77. $g(x) = x^3 + 2$ [1.5], [2.1]

78. $f(x) = 1 - x^2$ [8.7]

Find the domain of each function.

79. $f(x) = \dfrac{x - 3}{x + 7}$ [2.1]

80. $f(x) = \dfrac{x}{(x - 2)(x + 3)}$ [5.5]

81. $g(x) = \sqrt{10 - x}$ [7.1]

82. $g(x) = |x^2 - 6x + 7|$ [2.1]

SYNTHESIS

TW **83.** A student *incorrectly* reasons that

$$\log_b \frac{1}{x} = \log_b \frac{x}{xx}$$
$$= \log_b x - \log_b x + \log_b x = \log_b x.$$

What mistake has the student made?

TW **84.** Is it true that $\log_a x + \log_b x = \log_{ab} x$? Why or why not?

Express as an equivalent expression that is a single logarithm and, if possible, simplify. $\log_a (x^6 - x^4y^2 + x^2y^4 - y^6)$

85. $\log_a (x^8 - y^8) - \log_a (x^2 + y^2)$

86. $\log_a (x + y) + \log_a (x^2 - xy + y^2)$ $\log_a (x^3 + y^3)$

Express as an equivalent expression that is a sum or a difference of logarithms and, if possible, simplify.

87. $\log_a \sqrt{1 - s^2}$

88. $\log_a \dfrac{c - d}{\sqrt{c^2 - d^2}}$

Answers to Exercises 28–30, 33–46, 75–82, 87, and 88 are on p. IA-24.

89. If $\log_a x = 2$, $\log_a y = 3$, and $\log_a z = 4$, what is $\log_a \dfrac{\sqrt[3]{x^2 z}}{\sqrt[3]{y^2 z^{-2}}}$? $\frac{10}{3}$

90. If $\log_a x = 2$, what is $\log_a (1/x)$? -2

91. If $\log_a x = 2$, what is $\log_{1/a} x$? -2

Solve.

92. $\log_{10} 2000 - \log_{10} x = 3$ 2

93. $\log_2 80 + \log_2 x = 5$ $\frac{2}{5}$

Classify each of the following as true or false. Assume a, x, P, and $Q > 0$, $a \neq 1$.

94. $\log_a \left(\dfrac{P}{Q}\right)^x = x \log_a P - \log_a Q$ False

95. $\log_a (Q + Q^2) = \log_a Q + \log_a (Q + 1)$ True

96. Use graphs to show that

$$\log x^2 \neq \log x \cdot \log x.$$

(*Note*: log means $\log_{10}$.)

Try Exercise Answers: Section 9.4

7. $\log_3 81 + \log_3 27$ **13.** $\log_a (5 \cdot 14)$, or $\log_a 70$
17. $8 \log_a r$ **23.** $\log_2 25 - \log_2 13$ **27.** $\log_a \frac{17}{6}$
39. $4 \log_a x - 3 \log_a y - \log_a z$ **51.** $\log_a \dfrac{y^5}{x^{3/2}}$
55. 1.953 **65.** 7

Mid-Chapter Review

We use the following properties to simplify expressions and to rewrite equivalent logarithmic and exponential equations.

$\log_a x = m$ means $x = a^m$.	$\log_a a^k = k$
$\log_a (MN) = \log_a M + \log_a N$	$\log_a a = 1$
$\log_a \dfrac{M}{N} = \log_a M - \log_a N$	$\log_a 1 = 0$
$\log_a M^p = p \cdot \log_a M$	$\log x = \log_{10} x$

GUIDED SOLUTIONS

1. Find a formula for the inverse of $f(x) = 2x - 5$. [9.1]

Solution

$y = 2x - 5$

$\square = 2\square - 5$ Interchanging x and y

$\square = 2y$

$\dfrac{\square}{2} = y$ Solving for y

$f^{-1}(x) = \square$

1. $y = 2x - 5$
$x = 2y - 5$
$x + 5 = 2y$
$\dfrac{x + 5}{2} = y$
$f^{-1}(x) = \dfrac{x + 5}{2}$

2. Solve: $\log_4 x = 1$. [9.3]

Solution

$x = \square^{\square}$ Rewriting as an exponential equation

$x = \square$

2. $x = 4^1$
$x = 4$